

EXERCISE 1

Draw sketch graphs of the following:

- | | | |
|-------------------------|---------------------|-------------------|
| 1 $y = e^x$ | 2 $y = -e^x$ | 3 $y = \ln x$ |
| 4 $y = e^{-x}$ | 5 $y = 2 \ln x$ | 6 $y = \ln x + 2$ |
| 7 $y = e^{\frac{1}{2}}$ | 8 $y = e^{x+2}$ | 9 $y = \ln(x-3)$ |
| 10 $y = 2 \ln(x+1)$ | 11 $y = e^{2x} - 1$ | 12 $y = -e^{-x}$ |

- 13 a Using the same axes sketch the graphs of $y = e^{-x}$ and $y = x(2-x)$.
 b Use symmetry to write down the coordinates of the turning point of the curve $y = x(2-x)$.
 c How many solutions are there to the equation $e^{-x} = x(2-x)$?
- 14 The function $f(x) = \ln 2x$, $x > 0$.
 a Sketch $y = f(x)$, showing the point where the graph cuts the x -axis.
 b Find an expression for $f^{-1}(x)$.
 c Sketch the graph of $y = f^{-1}(x)$ and describe the relationship between the graphs of $y = f(x)$ and $y = f^{-1}(x)$.
- 15 On the same axes draw a sketch of the graph of $y = \ln x$ and a sketch of the graph of $y = \ln(x^3)$. Label your two curves clearly.
- 16 The functions $f(x)$ and $g(x)$ are defined by

$$f(x) = \ln x, \quad x > 0 \quad \text{and} \quad g(x) = x^2, \quad x \in \mathbb{R}.$$
 a Write down expressions for $fg(x)$ and $gf(x)$.
 b Show that $fg(x) = kf(x)$, for some constant k , and find k .
- 17 The function $f(x)$ is defined by

$$f(x) = 3e^x \text{ for all } x.$$
 a Sketch on the same axes $y = f(x)$ and $y = f^{-1}(x)$.
 b Find an expression for $f^{-1}(x)$.
 c State the domain and range of f^{-1} .
- 18 The function f is defined by $f(x) = 3 + \ln x$, $x > 0$.
 a State the range of the function f .
 b State the domain and range of the inverse function f^{-1} .
 c Obtain an expression for $f^{-1}(x)$.
 d The function g is defined for all x by

$$g(x) = ex^4.$$
 Show that $fg(x) = 4(1 + \ln x)$
 e Solve the equation $fg(x) = 16$.

Example 3

Solve the equation $\ln(3x - 1) = 4$.

$$\begin{aligned}\log_e(3x - 1) &= 4 \\ \text{So } 3x - 1 &= e^4 \\ x &= \frac{e^4 + 1}{3} \quad (= 18.53 \text{ to 2 decimal places})\end{aligned}$$

Example 4

Express $\log_{10}(x + 2) - \log_{10} x$ as a single logarithm.
Hence solve the equation $\log_{10}(x + 2) - \log_{10} x = 2$.

$$\begin{aligned}\log_{10}(x + 2) - \log_{10} x &= \log_{10}\left(\frac{x + 2}{x}\right) \\ \log_{10}(x + 2) - \log_{10} x &= 2 \\ \log_{10}\left(\frac{x + 2}{x}\right) &= 2 \\ 10^2 &= \frac{x + 2}{x} \\ 100x &= x + 2 \\ x &= \frac{2}{99}.\end{aligned}$$

EXERCISE 2

1 Simplify the following.

a $e^x \times e^x$

b $\frac{e^{8x}}{e^{2x}}$

c $(e^x)^3$

d e^0

e $\ln 2 + \ln x$

f $\ln 6 - \ln 2$

g $\ln x^3$

h $5 \ln 1$

i $\ln e$

j $e^2 \times e^x$

k $\ln \sqrt{e}$

l $\ln e^x$

m $\ln e^a$

n $(e^0) \times 7$

o $\frac{e^3 \times e^{-1}}{e}$

p $\ln(\ln e)$

2 Use a calculator to evaluate the following, giving the answer correct to two decimal places.

a $e^{2.5}$

b $\ln 10$

c $\frac{e^3 - 4}{7}$

d $\ln 12 - \ln 2$

e $\frac{\ln 18}{e^2}$

f $e^{-1} \times \ln 8$

3 Solve the equation $e^x = 7$, giving your answer correct to two decimal places.

In Questions 4 to 14 solve the equations.

4 $e^{2x} = 8$

5 $2e^x = 9$

6 $e^{x-1} = 11$

7 $e^{2x+1} = 4$

8 $e^{3x-1} = 8$

9 $e^{5x+2} = 100$

10 $e^{3x-1} = 1$

11 $\ln x - \ln 4 = 4$

12 $\ln x + \ln 2x = 12$

$$13 \ln x + \ln 3 - \ln 6 = \ln 10$$

$$14 \ln 2 + \ln x^2 = \ln 18$$

15 Solve the equation $2 \ln 2x = 1 + \ln 3$. Give your answer correct to two decimal places.

16 a Express $\ln(x+2) - \ln x$ as a single logarithm.

b Solve the equation $\ln(x+2) - \ln x = 4$.
Give your answer in terms of e .

17 Simplify the following.

a $\ln x^2 - \ln xy$

b $\ln x + \ln \frac{1}{x}$

c $\ln(\ln x^2) - \ln(\ln x)$

d $e^{\ln x}$

18 Solve the equations,

a $e^x \times e^{x+1} = 10$

b $\left(\frac{1}{e}\right)^{x+1} = 4$

c $e^{2x} - 3e^x + 2 = 0$

d $e^{\ln x} + \ln e^x = 8$

19 Solve the simultaneous equations $\ln(xy) = 5$

$[x > 0, y > 0]$

$\ln\left(\frac{x}{y}\right) = 1$

20 Find the value, or values, of x for which

$\ln(x+6) = 2 \ln(x-6).$

21 At time t minutes the temperature of a body in $^{\circ}\text{C}$ is

$5e^{-t} + 20.$

a Write down the temperature of the body at $t = 0$.

b Calculate the value of t when the temperature of the body reaches 22°C .
Give your answer correct to one decimal place.

22 Establish the formula $\log_a x = \frac{1}{\log_x a}.$

23 The positive numbers x, y and z are in geometric progression. Show that $\log x, \log y$ and $\log z$ are in arithmetic progression.

REVIEW EXERCISE 3

1 Draw sketch graphs of the following:

a $y = e^x + 1$

b $y = \ln x$

c $y = 2e^{-x}$

d $y = 3 \ln x$

e $y = e^{x-1}$

f $y = \ln(x-2), x > 2$

2 a If $f(x) = \ln x, x > 0$, write down an expression for $f^{-1}(x)$.

b Solve the equation $f^{-1}(x) = e$.

- 3 The function f is defined by $f(x) = 4 + \ln x$, $x > 0$. Obtain an expression for $f^{-1}(x)$.
- 4 The functions f and g are defined for all real values of x by $f(x) = 2x - 100$
 $g(x) = e^{-2x}$.
- State the range of g .
 - Find an expression for $g^{-1}(x)$.
 - On the same axes sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$.
 - Evaluate $fg(-2)$, giving your answer correct to 2 decimal places.
- 5 Given $f(x) = \ln(3 - 2x)$ and $g(x) = e^x$, evaluate $gf(1)$.
- 6 Sketch the graph of $y = \ln e^x - x$, $x \in \mathbb{R}$.
- 7 Show that the equation $x = \ln(x + 5)$ has a root between $x = 1$ and $x = 2$.
- 8 Solve the equations, giving your answers correct to 2 decimal places where necessary.
- $e^{3x} = 7$
 - $3e^x = 2$
 - $e^{x+1} = 5$
- 9 Solve the equations, giving your answers correct to 2 decimal places where necessary.
- $\ln(x + 1) = 2$
 - $\ln x - \ln 2 = 3$
 - $2 \ln 3x = 1$
 - $\ln x = \ln e^3$
- 10 The function f is defined by $f(x) = 5(2^{-x}) - 1$ for $x \geq 0$.
The graph of $y = f(x)$ meets the y -axis at $(0, a)$ and the x -axis at $(b, 0)$.
- State the value of a .
 - Find the value of b , correct to 2 decimal places.
 - Sketch the graph of $y = f(x)$.
 - Copy and complete the following statement:
'As $x \rightarrow \infty$, $f(x) \rightarrow \square$.'
- 11 Solve the equation $2 \ln 2x = 1 + \ln 3$. Give your answer in terms of e .
- 12 Solve the equation $\ln(x + 1) - \ln x = e$. Give your answer correct to 4 decimal places.
- 13 The temperature, $\theta^\circ\text{C}$, of an oven at time t minutes is given by
 $\theta = 300 + 50e^{-0.04t}$.
Find the value of θ when $t = 10$.
- 14 a By substituting $y = 2^x$ into the equation
 $2^{2x} - 5 \times 2^x + 4 = 0$
show that $y^2 - 5y + 4 = 0$.
- b Hence find the values of x which satisfy the equation $2^{2x} - 5 \times 2^x + 4 = 0$.

EXAMINATION EXERCISE 3

- 1 Solve the equation $e^{\ln x} + \ln e^x = 8$. [EDEXCEL]

- 2 Simplify:

a $\log x^2 - \log xy$ b $\log z + \log \frac{1}{z}$ c $\frac{\log x^3 - \log x}{\log x^2 - \log x}$ [OCR]

- 3 Express $\log(2\sqrt{10}) - \frac{1}{3}\log 0.8 - \log(\frac{10}{3})$ in the form $c + \log d$ where c and d are rational numbers and the logarithms are to base 10. [OCR]

- 4 Solve the equation

$$\ln(5x + 6) = 2 \ln(5x - 6) \quad \text{[EDEXCEL]}$$

- 5 It is given that $\ln x = p + 2$ and $\ln y = 3p$.

- i Express each of the following in terms of p :

a $\ln(xy)$, b $\ln(x^3)$, c $\ln\left(\frac{y}{e}\right)$.

- ii Express y in terms of x and e , simplifying your answer. [OCR]

- 6 The function f is defined by $f: x \mapsto e^x + k$, $x \in \mathbb{R}$ and k is a positive constant.

- a State the range of f .
b Find $f(\ln k)$, simplifying your answer.
c Find f^{-1} , the inverse function of f , in the form $f^{-1}: x \mapsto \dots$, stating its domain.
d On the same axes, sketch the curves with equations $y = f(x)$, and $y = f^{-1}(x)$, giving the coordinates of all points where the graphs cuts the axes.

[EDEXCEL]

- 7 The function f is defined by

$$f: x \mapsto \ln(5x - 2), x > \frac{2}{5}$$

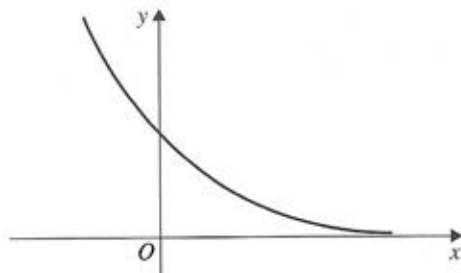
- a Find an expression for $f^{-1}(x)$.
b Write down the domain of f^{-1} .
c Solve, giving your answer to 3 decimal places,

$$\ln(5x - 2) = 2 \quad \text{[EDEXCEL]}$$

- 8 The diagram shows the graph of $y = f(x)$, where f is defined for all real numbers by

$$f(x) = 2e^{-x}.$$

- a Describe a sequence of geometrical transformations by which the above graph can be obtained from the graph of $y = e^x$.



- b Copy the above diagram and sketch on the same axes the graph of

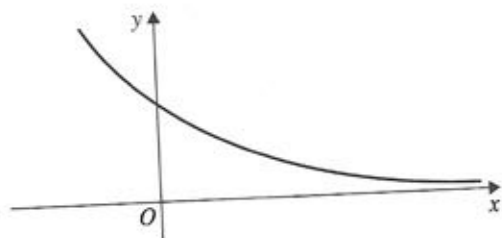
$$y = f^{-1}(x).$$

- c Find an expression for $f^{-1}(x)$.

- d State the domain and range of f^{-1} .

[AQA]

9 a



The diagram shows the graph of $y = f(x)$, where the function f is defined for all values of x by

$$f(x) = 5e^{-x}.$$

- i Write down the coordinates of the point where the graph intersects the y -axis.
 - ii State the range of the function f .
 - iii Find the value of $f(\ln 6)$, giving your answer as a fraction.
- b The function g is defined for all values of x by

$$g(x) = x + 10.$$

- i Show that $gf(x) = 5(e^{-x} + 2)$.
- ii State the range of the function gf .
- iii Sketch the graph of $y = gf(x)$.
- iv Show that $gf(x) = 11 \Rightarrow x = \ln 5$.

[AQA]

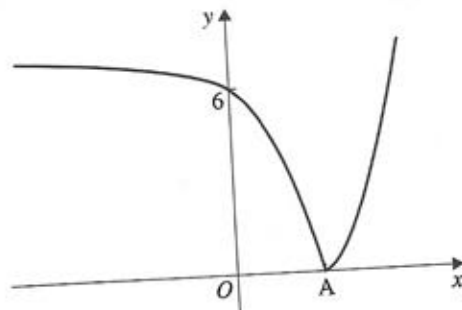
- 10 The functions f and g are defined for all real numbers by

$$f: x \mapsto 2e^{3x},$$

$$g: x \mapsto x - k,$$

where k is a positive constant.

- i Evaluate $fg(k)$.
- ii Find an expression for $f^{-1}(x)$.
- iii Sketch on the same diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$, indicating the relationship between the graphs.
- iv



The diagram shows the graph of $y = |gf(x)|$, which meets the x -axis at A and the y -axis at $(0, 6)$. Find k and determine the exact value of the x -coordinate of A .

[OCR]